"Interaction–free" interaction: entangling evolution via quantum Zeno effect

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The effect of entangling evolution induced by frequently repeated quantum measurement is presented. The interesting possibility of conditional freezing the system in maximally entangled state out of Zeno effect regime is also revealed. The illustration of the phenomena in terms of dynamical version of "interaction free" measurement is presented. Some general conclusions are provided.

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Interaction-free measurement (IFM) is one of the fascinating quantum phenomena. It has its origin in the Renninger [1] idea of "negative result experiment" Then the "interaction-free" concept has been considered by Dicke [2]. The ideas have been extended and modified by Elitzur and Vaidman (EV) in their well-known striking scheme [3] revealing strong modification of interference of the photon only by *presence* of the object. In their original scheme using one Mach-Zenhder interferometer in $\eta = 25\%$ of events photon gives us information about presence of the object - simbolised by ultrasensitive bomb - with *no interaction* with it. Surprisingly, the EV scheme can be improved to obtain the efficiency η to be arbitrarily close to unity. This result is due to Kwiat et. al. [4] who combined the idea of IFM with the quantum Zeno effect (QZE) which in another interesting, quantum phenomenon. In its original form [6] QZE relies on inhibition of the decay of unstable quantum system by frequently repeated measurement. After performance of the proposed experiment [7] there was a discussion on the essence of the QZE. While originally it has been explained using quantum collapse, now it is clear that it is *decoherence* with is the essential ingredient of the effect [8–10]. Subsequently the effect has been shown to imply the broad class of different physical phenomena [11], [12], [13], [14], [15].

The IFM scheme presented in Ref. [4] involves two cavities connected via weakly transparent mirror. This results in a vary small probability that after one round trip time a photon being initially in left cavity will be found in right one. But quantum coherent photon evolution allows for cumulation of the effect of transmission from left to right (see [4] . Hence after the time T being sum of large number of round trip times the photon will completely leave the left initial cavity moving to the right one giving finally the probability $p = 0$ of finding it in the initial cavity. Note that the intermediate states of the photon during the evolution are *superpositions* of two photon states "present in left cavity", say, $|0\rangle_f$ and "present in right cavity", say , $|1\rangle_f$.

The situation dramatically changes when one puts the absorbing object in the right cavity as the evolution of the photon is *no* longer coherent. After any round trip time Δt the superposition is destroyed - the photom is, in a sense, forced to decide whether it is in left or right cavity. This is nothing but quantum-mechanical measurement. Here the measurement is very frequent if compared with the time T as $\Delta t << T$. It leads to QZE so that the evolution corresponding to transition from left to right is almost stoped - after time T the photon can be found with probability $p \simeq 1$ in *left* cavity. If one consider the value of p in the absence (presence) of the object it is clear that testing the presence of the photon in left cavity one can almost with certainity check whether there is an absorbing object in right cavity although the photon *never* interacted with the object in a sence that it has been neither absorbed or scattered. The original EV proposal [3] to consider the object as an ultrasensitive bomb, which explodes if only one photon is scattered on it, makes the latter effect even more striking. As showed in [4] the presence of the object (bomb) can be simulated by the atom which can be in one of two levels $|0\rangle$, $|1\rangle$. The third level $|2\rangle$ has a very short life time, after which it decays to the ground state $|0\rangle$ emitting than the photon with some frequency ω_{signal}. Now, if the frequency of the photon ω_f is adjusted to the transition between states $|1\rangle$, $|2\rangle$ i.e. if one has $\omega_{|1\rangle\to|2\rangle} = \omega_f$ then the atom in the state $|0\rangle$ ($|1\rangle$) corresponds to the absence (presence) of the bomb in the left cavity.

In this paper we consider the case when the "quantum bomb" i.e. the atom evolves coherently form state $|0\rangle$ to $|1\rangle$. Subsequently we present two effects. First, we consider the simple dynamical version of IFM from Ref. [4]. In this case, as in original proposal, the photon is almost never absorbed, so we have not iteraction in this sense. But we show that now the atom becomes periodically entangled and disentangled with the photon - so the price for lack of interaction in one sense (no absorbtion) is payed via another kind of interaction (entanglement).

Second, we study the case when QZE regime is abondoned. It can be done in two ways:

(i) One can wait much longer then the scale of the characteristic time of the photon evolution T. The time Δt between measurements is much more less then T but it is *finite*. This implies that if we wait much longer than

T the approximation $p \simeq 1$ will no longer be true.

(ii) One can make the measurements in the effect *not* to be frequent (then $\Delta t << T$ is false). It would correspond to the fact that the transparency of the mirror between cavities is no longer small.

We show that in both cases the system can be with a great probability *conditionally frozed* . This effect can be described as follows. Suppose that we have the photon in left cavity, and the atom in the ground state $|0\rangle$ i.e. that our system is in the *product* state, and we allow to evolve both of them. The transmitivity of the mirror between the cavities is not necessarily small and we are able to find whether atom absorbed photon detecting then the emitted ω_{signal}. As we shall see below, it appears, that if we wait for a long time then with probability 50% no detectors will click and our atom + photon system will stop their evolution being frosen in a maximally entangled state. We call this effect the conditional forsing, as it is conditioned by the negative detection result. But we see that such a frosing can occur with quite big probability. At the end of the paper we give simple explanation of the origin of the effect and formulate some general conjecture.

The photon + atom system will be described using the tensor product space $\mathcal{H} = \mathcal{H}_f \otimes \mathcal{H}_a$ where the space $\mathcal{H}_f$ ($\mathcal{H}_a$) is spanned by vectors $|0\rangle_f, |1\rangle_f$ ($|0\rangle, |1\rangle$). Their tensor products define the product basis in $\mathcal{H}$. Any two component system which can be described in such a way is usually called 2×2 or two-qubit system and is paradigmatic in investigations of quantum entanglement. Sometimes we shall use maximally entangled states

$$\Psi_{\pm} = \frac{1}{\sqrt{2}}(|0\rangle_f|1\rangle \pm |1\rangle_f|0\rangle) \tag{1}$$

The evolution of the photon can be schematically represented by

$$\phi_f(t) = e^{i\sigma_y t}\phi_0 \tag{2}$$

with σ_y being second Pauli matrix written in the basis $\{|0\rangle_f, |1\rangle_f\}$. Although, in general, the evolution can be more complicated, the simple picture (2) allows us to explain the essence of the effects which will take place. In particular in the above evolution the time T after which photon being initially in left cavity (state $|0\rangle_f$) with certainity reaches empty right cavity (state $|1\rangle_f$) is fixed and simply amounts to $T = \frac{\pi}{2}$. It is useful here we shall be interested whether times are small or big *relative to time* T.

Further we assume the evolution of the atom to be governed also by the simple Hamiltonian $-\sigma_y$. If we had $\omega_{|1\rangle\to|2\rangle} \neq \omega_f$ we would have no measurement and our system would subject to the product evolution

$$e^{iHt} = e^{i\sigma_y t} \otimes e^{-i\sigma_y t}, \quad \text{with} \;\; H = \sigma_y \otimes I - I \otimes \sigma_y \tag{3}$$

Note that in this case the time $T = \frac{\pi}{2}$ is the moment when both photon and atom meet each other in the right cavity in the state $|1\rangle_f|1\rangle$. In our case this state corresponds to the "explosion" state $\Psi_{exp} = |1\rangle_f|1\rangle$ becouse we require, as in [4], $\omega_{|1\rangle\to|2\rangle} \neq \omega_f$. Then, if the photon is in the right cavity (state $|1\rangle_f$) and at the same time atom is in the state $|1\rangle$, the photon is absorbed and then after rapid decay the signal photon is emitted. This effect, simbolising an "explosion" corresponds to the quantum measurement checking whether the system state is orthogonal to the product state $|\Psi_{exp}\rangle$. The corresponding observable is $P^{\perp} = I - |\Psi_{exp}\rangle\langle\Psi_{exp}|$. The above measurement will occur after any round trip time Δt. Both Δt as well as T can be changed by maneuvering with the size of the cavities, the radiofrequency driving the atom and the transmitivity of the mirror. In our picture both times are simply rescaled by fixing $T = \frac{\pi}{2}$.

What will be the evolution of our system if the measurements is frequent in the sense that $\Delta t << T$?

Before we answer the question let us recall some characterisation of QZE dynamics. It is known (see [16]) that for any bounded Hamiltonian H and projector P we have the limit

$$lim_{n\to\infty}(Pe^{iHT'/n}P)^n = Pe^{iPHPT'} \tag{4}$$

Any initial state Ψ_0 subjects the above unitary transformation with the probability $\langle\Psi_0|P|\Psi_0\rangle$. Now one can consider any evolution e^{iHt}, with H bounded, after any Δt interrupted by the measurement of observable P. It can be argued that if the measurement is frequent then, during the time period (t_0, T) with $t_0 >> \Delta t$, the formula (4) allows to approximate such interrupted evolution by by the *new* one taking place in subspace $\mathcal{H}' = P\mathcal{H}$

$$U_{cut}(t) = e^{iH_{cut}t}, \text{ with } H_{cut} = PHP \tag{5}$$

According to the remark after formula (4) the above form of dynamics holds for all initial states Ψ_0 such that $P^{\perp}\Psi_0 = \Psi_0$. Summarising - in the limit of very frequent measurements the system is confined in the subspace $P\mathcal{H}$ subjecting the drastically different evolution (5).

Coming back to our photon + atom system we put $P^{\perp}$ in place of P . We also assume that T' includes several $T = \frac{\pi}{2}$ periods, say $T' = 5\pi$ or so. As in original IFM scheme we consider frequent measurement case which means $\Delta t = \frac{\pi}{2n} << T \sim T'$. If initial state of our system belongs to the subspace $\mathcal{H}^{\perp}$ spanned by three vectors $|0\rangle_f|0\rangle$, $|0\rangle_f|1\rangle$, $|1\rangle_f|0\rangle$ then the limit of large n the dynamics is given by the new Hamiltonian $P^{\perp}(\sigma_y \otimes I - I \otimes \sigma_y)P^{\perp}$. It can be easily calculated that it generates the following limit unitary evolution in subspace $\mathcal{H}^{\perp}$

$$U_{lim}(t) = \begin{bmatrix} \cos(\sqrt{2}t) & -\frac{1}{\sqrt{2}}\sin(\sqrt{2}t) & \frac{1}{\sqrt{2}}\sin(\sqrt{2}t) \\ \frac{1}{\sqrt{2}}\sin(\sqrt{2}t) & \cos^2(\frac{\sqrt{2}}{2}t) & \sin^2(\frac{\sqrt{2}}{2}t) \\ -\frac{1}{\sqrt{2}}\sin(\sqrt{2}t) & \sin^2(\frac{\sqrt{2}}{2}t) & \cos^2(\frac{\sqrt{2}}{2}t) \end{bmatrix} \tag{6}$$

QZE causes that our system is confined in the subspace orthogonal to the "explosion" state $|1\rangle_f|1\rangle$. Thus, as in original IFM effect the frequent *possibility* of "explosion" prevents from *actualisation* of it. But here this possibility, alhough *never* actualised, strongly modifies the evolution of the *whole* system including the atom. In fact, if the initial state of the system is the product one $|\psi_0\rangle = |0\rangle_f|0\rangle$ belonging to $\mathcal{H}^\perp$ then it evolves as $|\psi(t)\rangle = U_{lim}(t)|\psi_0\rangle = \cos(\sqrt{2}t)|0\rangle|0\rangle + \sin(\sqrt{2}t)\Psi_-$. Thus the system evolves from the product state ψ_0 to the *maximally entangled* state Ψ_- !. So the frequent possibility of "explosion" results also in highely *entangled* evolution. The process is "interaction free" in one sense (no absorbtion of the photon), but the cost for it must be paid resulting in strong interaction in the other sense (entanglement).

According to undisturbed product evolution (3) after time $T = \frac{\pi}{2}$ photon and atom should meet each other in the "explosion" state $|\Psi_{exp}\rangle$. Here, however, they are in highely entangled state $|\psi(\frac{\pi}{2})\rangle$ still orthogonal to $|\Psi_{exp}\rangle$. It is worth to note that the maximum of entanglement comes at time $\frac{T}{\sqrt{2}} < T$ hence it happens before the moment of the supposed meet at the "explosion" state $|\Psi_{exp}\rangle$.

We see that in the process we have in general two time scales. One of them due to Δt describes the frequency measurements interrupting evolution or, in other words, the priods of product evolution. The another one, represented by the interval (t_0, T'), will be called the scale of Zeno effect. It describes us the region where the Zeno effect approximation i.e. the formula (6) is good. What happens, however, if we abandon this scale passing to the large times $t >> T'$? More precisely, what happens if we keep Δt fixed and take the limit $t \to \infty$? In general the problem is complicated. However we can specify our problem as follows. Assume that we can somehow detect the signal photon ω_{signal}. No detection up to time t means that the atom have not already absorbed the probing photon (the one with ω_f). Let us ask two questions: (i') is there a nonzero probability that we have never detected the signal photon up to very large time ? (ii') if so, what would be the form of conditional evolution of our atom + photon system then ? To answer those questions let us ask about existence of the limit

$$\lim_{n\to\infty}(P^\perp e^{iH\Delta t}P^\perp)^n = \lim_{n\to\infty} W(\Delta t)^n \tag{7}$$

where abbreviation $W(\Delta t)$ has been introduced. Belowe we shall see that the above limit exists and posses property which leads to an interesting effect.

From further analysis we exclude the periods $\Delta t \neq k\pi, \frac{k\pi}{2}$ because for them the measurements commutes with the product evolution (3) having then no impact on it. Note that those are *the only* assumptions about Δt. Here the measurement no longer need to be frequent if compared with $T = \frac{\pi}{2}$.

To see what happens we write it in the new *entangled* basis:

$$|\Psi_1\rangle = \Psi_+, \quad |\Psi_2\rangle = \alpha((2/\tau)|0\rangle_f|0\rangle + \sqrt{2}\Psi_-)$$
$$|\Psi_3\rangle = \beta(-\tau|0\rangle_f|0\rangle + \sqrt{2}\Psi_-), \quad |\Psi_4\rangle = |1\rangle_f|1\rangle \tag{8}$$

with $\tau = \tan(\Delta t)$, $\alpha = \frac{|\tau|}{\sqrt{2\tau^2+4}}$, $\beta = \frac{1}{\sqrt{\tau^2+2}}$. If we restrict to the subspace $\mathcal{H}'$ then the limit (7) can be written in the form :

$$\lim_{n\to\infty} W(\Delta t)^n = \lim_{n\to\infty} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\sin\phi & -\cos\phi \\ 0 & \delta\cos\phi & -\delta\sin\phi \\ 0 & 0 & 0 \end{bmatrix}'^n . \tag{9}$$

Here $\sin\phi = \frac{\tau^2-2}{\tau^2+2}$, $\cos\phi = \frac{4\tau}{\sqrt{2(\tau^2+2)}}$. The of coefficient $\delta = \cos^2(\Delta t)$ is strictly lesser then 1 for we assumed $\Delta t \neq k\pi$. The prime at the matrix in (9) is to stress that it is written in the new *entangled* basis (8).

It is no difficult to show that the sequence of 2×2 submatrix

$$A^n = \begin{bmatrix} \sin\phi & \cos\phi \\ \delta\cos\phi & -\delta\sin\phi \end{bmatrix}^n \tag{10}$$

with $\delta < 1$ tends to zero matrix. To see it consider the matrix norm $||C|| = \underset{x}{max}||Cx||$ where maximum is taken over all vectors with norm $||x|| = 1$. It has the properties (a) $||AB|| \leq ||A||||B||$ and (b) $||C^\dagger C|| = ||C||^2$. To show that $\lim_{n\to\infty} A^n = 0$ it only suffices to prove that $\lim_{n\to\infty}||A^{2n}|| = 0$. From the property (a) of the matrix norm we have $||A^{2n}|| \leq ||A^2||^n$ and $||A^2|| = \sqrt{||(A^T)^2A^2||}$ Hence we only need to show that the norm of the new matrix $B = (A^T)^2A^2$ is strictly less than 1 i. e. we need to show that $||B|| < 1$. The original matrix A can be written as $A = A_\delta O$ where $A_\delta = diag(1, -\delta)$ and O is a two-dimensional rotation around axis x about the angle $\phi + \pi$. Then (a) we have $||B|| \leq ||A||^4 \leq ||A_\delta||^2||O||^2 = (max[1,\delta])^2 \cdot 1 = 1$ as O is rotation not changing the norm and the norm of any hermitian operator is equivalent to maximum of modulus of its eigenvalues. Thus we have established that $||B|| \leq 1$. Note that B is hermitian and has positive eigenvalues as it is of the form C^TC. Hence $||B|| = max(b_1, b_2)$ where b_1,b_2 are eigenvalues of B. We already know that none of them is greater then 1. Can any of them be equal to unity ? The answer is negative. Indeed, we can calculate explicitly both trace and determinant of B resulting in $b_1b_2 = detB = \delta^4$ and $b_1 + b_2 = Tr(B) = \sin^2\phi(1+\delta^4) + 2\delta^2\cos^2\phi$. Assumption that some of eigenvalues is equal to 1 gives us, the

equation $(1-\delta^4)\cos^2\phi = 0$. As $\delta < 1$ it would imply that $\cos\phi = 0$ but it evidently contaradict the assumption $\Delta t \neq \frac{\pi}{2}$. Thus both b_1, b_2 must be less then unity and then $||B|| = max(b_1, b_2) < 1$. According to former remarks it implies $\lim_{n\to\infty} A^n = 0$.

But what it means for the dynamics of our system ? The result is quite interesting. In fact we have

$$\lim_{n\to\infty} W(\Delta t)^n = diag[1,0,0]' = |\Psi_+\rangle\langle\Psi_+| \tag{11}$$

It means that under the condition that the negative result of our measurement of $P^\perp$ have not occurred our system is frozen in maximally entangled state despite the fact that it has seemingly enough room to move: three dimensions in the presence of only one of the orthogonal subspace spanned by vector Ψ_{exp}.

What is the probability of the process ? It depends on the initial state of the system Ψ_0. It can be easily verified that the probability of staying of the state in the subspace $P^\perp\mathcal{H}$ after n measurements is $p(n\Delta t) = ||W(\Delta t)^n\Psi_0||^2$ Taking the limit we obtain simply that $p(\infty) = |\langle P_+|\Psi_0\rangle|^2$. It is interesting to examine one example. Consider our photon plus atom system in the initial product i.e. *disentangled* state $|0\rangle_f|0\rangle$ It means that photon is initially in left cavity and atom is in the ground state. Let add the detectors to the scheme waiting for possible signal photon. Then there is quite large probability $p(\infty) = \frac{1}{2} = |\langle\Psi_+|0\rangle_f|0\rangle|^2$ that none detector will fire and that the evolution of our system will gradually stop, frozing finally photon *maximally entangled* with the atom.

The important remark should be given here. The above process of *conditional* frozing the state is *out* of Zeno effect regime also in the sense that, unlike in original scheme [4], it does not require Δt to be small. Hence the transmitivity of the mirror in the present scheme need not be small too. This effect (as well as the previous one) could be implemented in other schemes, for example, those involving QED cavities.

It is illustrative to explain the origin of the conditional frosing. It is immediate to see that the surviving state Ψ_+ is the only state which is invariant with respect to both the involved measurement and the product evolution (3). So the limit of large times simply sweeps out all the states which are not of this kind. Following this observation one can expect that the conditional frosing is quite *general* phenomena occuring in unitary evolution periodically interrupted by uncomplete von Neumann measurement. The only important requirements seem to be the irreducibility of the evolution with respect to the subspaces defined by the measurement and the existence of states being invariants of both evolution and the measurement.

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